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INVESTIGATION ON RESISTANCE AND HEAT TRANSFER OF TURBULENT AIR FLOW IN AXISYMMETRICAL CHANNELS WITH LONGITUDINAL PRESSURE GRADIENT

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Аннотация—В статье излагаются результаты экспериментальных исследований характеристик турбулентного пограничного слоя при течении нагретого воздуха в осесимметричных диффузорных и конфузорных каналах с охлаждаемыми стенками. Экспериментальная установка, на которой проводились исследования, позволила установить влияние продольного градиента давления и интенсивности охлаждения на характеристики динамического и теплового пограничных слоёв. Обработка и обобщение опытных данных проведены в параметрической форме. Найдены зависимости формпараметров, характеризующих форму профиля скоростей и температур, а также определяющих закон трения и теплообмена, от продольного градиента давления. Эти зависимости представлены в виде расчётных уравнений.

Полученный обширный экспериментальный материал позволил разработать методы расчёта турбулентного пограничного слоя при течении газа с продольным градиентом давления и теплообменом, которые дают возможность определить толщины потери импульса и энергии, толщины динамического и теплового вытеснения, касательное напряжение и удельный тепловой поток на стенке.

В статье приводятся кроме того результаты экспериментального исследования распределения касательного напряжения и удельного теплового потока, а также коэффициентов турбулентного обмена импульсов и тепла по сечению турбулентного пограничного слоя при градиентном течении газа.

NOMENCLATURE

θ, thickness of impulse loss;
φ, thickness of energy loss;
δ^*, thickness of displacement;
Δ^*, thickness of heat displacement;
u_1, velocity in potential flow;
p_1, static pressure in undisturbed flow;
T_w, wall temperature;
q_w, heat flow on a wall;
τ_w, tangential friction stress on a wall;
r, radius (distance from surface of axisymmetrical channel to its axis) in considered section;
c_p, mass specific heat capacity at constant pressure;
T_0, deceleration temperature in considered point of a boundary layer;
T_{00}, deceleration temperature of a potential flow;
T_{01}, equilibrium temperature of a potential flow;
β, angle between normals dropped to a wall and channel axis in plane of control section;
δ, thickness of dynamic boundary layer;
Δ, thickness of heat boundary layer;
ν_1, kinematic viscosity of potential flow;
u, v, normal and tangential components of velocity;
x, y, co-ordinates, directed both along and normal to a flow respectively;
$\bar{x}$, dimensional co-ordinate;
A_τ, coefficient of turbulent heat transfer of impulses;
A_q, coefficient of turbulent transfer of heat.

1. INTRODUCTION

THE study of turbulent boundary layer properties with longitudinal pressure gradients is of great practical importance, both for developing methods of calculating friction and heat transfer in modern industrial installations and for discovering the mechanism of turbulent mixing under these conditions.

In published works on the dynamic boundary layer investigation of the boundary with large pressure gradients has been mainly considered [1–4]. The known semi-empirical methods of calculating an isothermal turbulent boundary layer in the region of the large pressure gradients allow one to calculate fairly reliably the dynamic properties of a turbulent boundary layer and to determine the position of the separation point of the boundary layer. However, the question of the extension of these methods to turbulent boundary layer calculations for intensive heat transfer is a modern problem. Methods of calculating a heat boundary layer, which are applicable more to engineering practice, and are based on the application of a similarity of heat and momentum transfer processes as well as the similarity of laws of friction and heat transfer in regions of large positive pressure gradients, are inapplicable here [5, 6]. Several works are known in which there has been a theoretical attempt to take into account the influence of large pressure gradients upon the properties of dynamic and heat boundary layers [7, 8]. However, the great number of poorly grounded assumptions and the unjustified clumsiness of design formulae reduce the practical value of these investigations. Since in the immediate future one can not rely on positive achievement in applying static theory to the study of anisotropic turbulence, one must acknowledge comprehensive experiments as the most effective method of investigation.

The above-mentioned considerations served as a basis for investigating turbulent boundary layers in a compressible gas with considerable longitudinal pressure gradients and intensive heat fluxes.

2. EXPERIMENTAL INSTALLATION. MEASURED VALUES AND MEASURING INDICATORS

The turbulent boundary layer properties were investigated for motion of heated air in axisymmetrical diffusers with aperture angles of 8°4′ and 12°, and in a confuser with a contraction angle of 8°. Experiments were carried out in a range of numbers $Re = (1{\cdot}688\text{–}8{\cdot}48) \times 10^5$. The wall temperature of the channels of experimental sections, cooled by water, ranged from 286° to 320°K, and the temperature of the air flow from 425° to 623°K. The velocities of the potential flow varied within wide limits up to $M = 0{\cdot}5$.

Each experimental portion of the channels consisted of five heat-insulated parts with double walls forming a space through which cooled water was slowly passed. The internal wall 2 mm thick was made of copper; the external wall 2·5 mm thick was of steel. Each part was 150 mm long. The internal diameter of the diffusers was 115 mm at the air entrance and 220 and 272·5 mm at the air exit. The internal diameter of the concentrator was 145 mm at the air entrance and 45 mm at the air exit. The cooled water was supplied to each part from a general pressure tank. The heated water flowed into a drain. The volume of water flowing through each part was kept constant. The temperature of the water in various parts was measured by differential chromel–copel thermocouples.

The experimental portions were component parts of the installation which comprised an open wind tunnel of intermittent operation. The installation consisted of the following parts: an air compressor; the receiver 3·5 m³ in capacity; an electric-heater 300 kilowatt in power; an injector; a changeable experimental portion; some temporary, connecting and stabilizing portions with metal lattices inserted which served to level the fields of velocities and temperatures.

While testing, the following values were measured: the air pressure in front of the electric-heater and in front of the working nozzle; the air temperature at the exit from the heater; static and complete pressures as well as temperatures in sections of each part at 30 mm from the joint plane between the parts; the static pressure in two intermediate sections (in the direction of air flow) in which the pressure gradient reached the highest values; the temperatures of the internal wall surface of a channel in the same sections in which velocities, temperatures and static air pressures were measured; the temperature drop and the volume of cooling water in all parts; the pressure drop in a measuring diaphragm.

Total pressure in the boundary layer and in undisturbed flow in sections before the cooled parts was measured by pneumo-metric microtubes of complete pressure, connected to a differential manometer filled with ethyl alcohol, water or mercury.

Static pressure was measured with the help of a selection tube of static pressure, inserted into the channel through its wall. This tube, as well as the open end of the Pitot tube, was in exactly the same plane as the channel. A critical point of the Pitot tube was at 30 mm from the joint plane between the parts. The other end of the selection tube of static pressure was adjusted to the differential manometer, the second tube of which communicated with the atmosphere.

Flow temperatures in sections of the working portion were measured by movable chromel–alumel micro-thermocouples in equivalent positions to the parts in which static and dynamic pressures were measured.

A record of air temperatures in all the sections was made on the band of an electronic balancing band potentiometer.

The Pitot tubes and micro-thermocouples were installed using micrometer screws which allowed the measurement of dynamic pressures and temperatures in entry sections of the parts of the working portion at intervals of 0·05 mm. Each part was equipped with two micrometer screws. It was therefore possible to measure the dynamic pressures and temperatures simultaneously at corresponding points of a boundary layer in control sections of all the five parts of the working portion.

3. METHODS OF TREATING THE EXPERIMENTAL DATA

Diagrams for the distribution of velocity and temperature in the boundary layer of each part were plotted using measurements of complete pressures and temperatures for flow in sections of the boundary layer in all parts of the working portions, and for all static pressures in these sections. These sections determine the integral properties of the boundary layer: the thickness of impulse loss θ, that of displacement δ^*, that of energy loss φ, and that of a heat displacement Δ^*. Diagrams of the changes in these properties were then plotted against the length of the experimental portion (along the x co-ordinate). Moreover, diagrams of velocity, temperature and density changes (u_1, T_{01} and ρ_1) as well as those of wall temperature T_w in the undisturbed flow were plotted against the length of the channel. These diagrams were used to calculate the tangential stress τ_w and the heat flow q_w at the walls of channels by integral ratios of impulses and energy for a boundary layer.

For the axisymmetrical gas flow considered in our work, the integral ratios have the following form:

(a) for impulse transfer

$$\frac{d\theta}{dx} + \left(\frac{H+2}{u_1}\frac{du_1}{dx} + \frac{1}{r}\frac{dr}{dx} + \frac{1}{\rho_1}\frac{d\rho_1}{dx}\right)\theta = \frac{\tau_w}{\rho_1 u_1^2} \tag{1}$$

(b) for energy transfer

$$\frac{d\varphi}{dx} + \left[\frac{1}{u_1}\frac{du_1}{dx} + \frac{1}{r}\frac{dr}{dx} + \frac{1}{T_{01}-T_w}\frac{d(T_{01}-T_w)}{dx} + \frac{1}{\rho_1}\frac{d\rho_1}{dx}\right]\varphi = \frac{q_w}{c_p\rho_1 u_1(T_{01}-T_w)} \tag{2}$$

where

$$\theta = \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right)\left(1 - \frac{y\cos\beta}{r}\right) dy$$

$$H = \frac{\delta^*}{\theta};\ \delta^* = \int_0^{x\delta}\left(1 - \frac{\rho u}{\rho_1 u_1}\right)\left(1 - \frac{y\cos\beta}{r}\right) dy;$$

$$\varphi = \int_0^\Delta \frac{\rho u}{\rho_1 u_1}\left(\frac{T_{00}-T_0}{T_{01}-T_w}\right)\left(1 - \frac{y\cos\beta}{r}\right) dy.$$

u, velocity in direction of axis y;
x, distance parallel to a wall.

The values of the static density ρ and ρ_1 were calculated from the state equation $p = g\rho RT$ for experimental temperatures T and T_1 (where T and T_1 are thermodynamic temperatures at the point of the boundary layer considered, and at its external boundary).

The reliability of the determination of local values of τ_w or the local friction coefficient

$$c_f = \frac{2\tau_w}{\rho_1 u_1^2}$$

and specific heat flow q_w using the method mentioned depends essentially upon the accuracy of the graphical determination of the initial experimental parameters. Therefore, the local values of c_f and q_w at the surface of a channel were still determined by two independent methods.

The method of determining c_f was based on the well-known fact that the universal logarithmic velocity profile which for an isothermal flow is described by equation (3) is preserved in the inner region of a turbulent core in the boundary layer:

$$\frac{u}{v_{*w}} = 5{\cdot}75 \log \frac{y v_{*w}}{\nu} + 5.5 \qquad (3)$$

where $v_{*w} = \sqrt{\tau_w/\rho}$ and is the dynamic velocity at the wall; ν is the kinematic viscosity.

As is known, law (3) is obtained by integrating the Prandtl equation for the tangential stress:

$$\tau = \rho l^2 \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 \qquad (4)$$

for $\rho = \text{const}$; $l = \kappa y$; $\tau = \tau_w$; the determination of the integration constant and the constant κ is done by measurements of the velocity distribution in a smooth tube made by Nikuradse.

In order to obtain the universal law for the velocity distribution of the turbulent core in a boundary layer at the gas flow, it is necessary to allow for a density change with temperature while integrating equation (4) and to make a corresponding correction for heat transfer when determining the integration constant.

Integrating equation (4) allowing for a change of the gas density ($l = \kappa y$; $\tau = \tau_w$), we get:

$$\frac{1}{\kappa} \ln y = \frac{1}{v_{*w}} \int \frac{\mathrm{d}u}{\sqrt{\rho_1/\rho}}. \qquad (5)$$

As our measurements in a region near a wall showed, we may assume that:

$$\frac{T - T_w}{T_{01} - T_w} = \frac{u}{u_1}. \qquad (6)$$

Applying equation (6), equation (5) is reduced as follows:

$$\frac{1}{\kappa} \ln y = \frac{1}{\sqrt{c_f/2}} \left[\frac{2\sqrt{[\bar{T}_w + (1 - \bar{T}_w)(u/u_1)]}}{1 - \bar{T}_w} + c\right]. \qquad (7)$$

Taking $\kappa = 0{\cdot}4$ and determining the integration constant c, as has been done [8] assuming a close approximation between the turbulent and laminar velocity distributions in the immediate proximity of a wall where a laminar and turbulent tangential stress are of the same order, we obtain:

$$2\left[\frac{\sqrt{[\bar{T}_w + (1 - \bar{T}_w)(u/u_1)]} - \sqrt{\bar{T}_w}}{1 - \bar{T}_w}\right] = [5{\cdot}75 \log (Re_y \sqrt{c_f/2}) + 5{\cdot}5] \sqrt{c_f/2} \qquad (8)$$

where

$$\bar{T}_w = \frac{T_w}{T_{01}}; \quad Re_y = \frac{u_1 y}{S}.$$

Plotting the profile of velocities (8) for given $\bar{T}_w$ with variables $u/u_1 = f(y/\delta)$, we get a lattice of curves at fixed values of c_f. If we plot the experimental values for velocities for a particular section at corresponding values of $\bar{T}_w$ on such a diagram, then where the logarithmic part of the velocity profile intersects one of lattice curves, we find the corresponding value of c_f, which will be the local value of the friction coefficient for the section of the wall considered.

The experimental values obtained in our experiments for different values of the form parameter of the pressure gradient Γ, as well as the curve corresponding to the universal logarithmic law of velocity distribution, is given in Fig. 1.

$$\Gamma = \left(\frac{\theta}{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x}\right) Re_\theta^{1/4} = \left(-\frac{\theta}{\rho_1 u^2}\frac{\mathrm{d}p}{\mathrm{d}x}\right) Re_\theta^{1/4} \qquad (9)$$

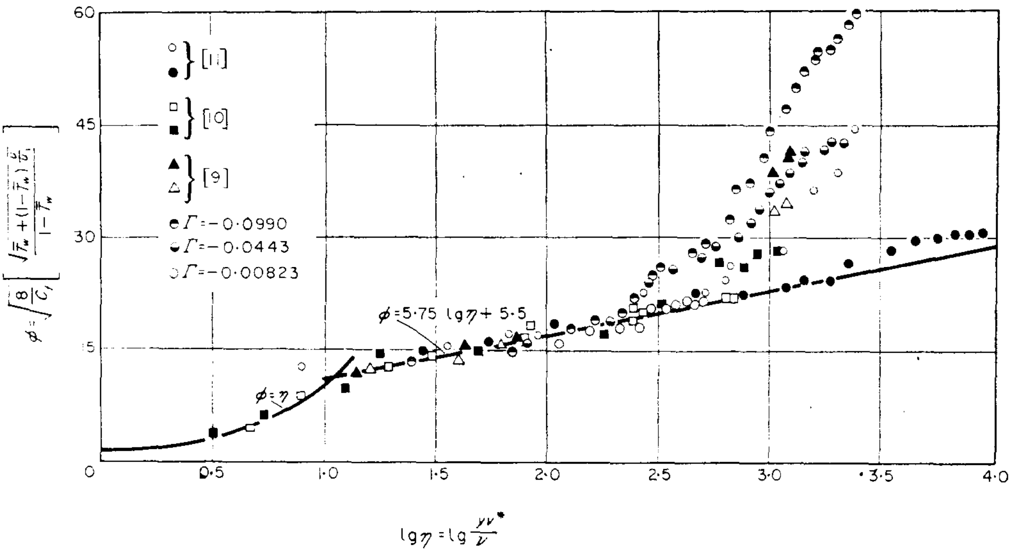

FIG. 1. The universal distribution of velocities in a boundary layer for positive pressure gradient.

where $Re_\theta = (u_1\theta/\nu_1)$; ν_1 is the kinematic viscosity of a potential flow in the section considered.

From this diagram it can be seen that the universal logarithmic profile of velocities is approximately preserved for one third of the thickness of a boundary layer. This conclusion is confirmed by experiments made by Brebner and Bagley [9], Fage [10], Schubauer and Klebanoff [11]. Note that outside the limits of the fully turbulent part of the layer near the wall, the divergence of experimental points from the logarithmic profile of velocities is greater the higher the pressure gradient.

An analogous diagram was plotted for distributing temperatures. The experimental profiles of temperatures are given in Fig. 2. From this diagram it can be seen that the universal law of the temperature distribution is obeyed over a greater portion of the boundary layer thickness than is the law of velocity distribution. Consequently, this diagram shows that the pressure gradient does not greatly influence the temperature distribution in a boundary layer.

The method of heat balance was the second means of determining specific heat flow. Specific heat flows on a wall were determined by measuring the volume of cooling water and temperature of water in parts of the experimental portion.

In order to determine the distribution of τ and

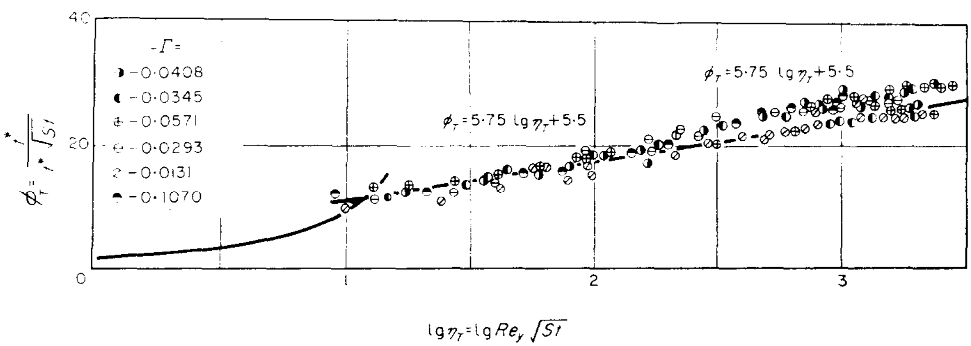

FIG. 2. The universal distribution of temperatures in a boundary layer for positive pressure gradient.

q for the section of the boundary layer, differential impulse and energy equations were integrated with regard for the continuity equation up to a current value of the co-ordinate y.

The initial equations are written in the following form:

impulse,

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{1}{r}\frac{\partial(\tau r)}{\partial y}, \quad (10)$$

continuity,

$$\frac{\partial(r\rho u)}{\partial x} + \frac{\partial(r\rho v)}{\partial y} = 0, \quad (11)$$

energy,

$$\rho u \frac{\partial T_0}{\partial x} + \rho v \frac{\partial T_0}{\partial y} = \frac{1}{c_p r}\frac{\partial(qr)}{\partial y}, \quad (12)$$

where x, y are co-ordinates, directed along the flow and normal to it, respectively. u and v are the normal and tangential components of the velocity.

Integrating equations (10) and (12) with regard for equation (11) for the thickness of a boundary layer from zero to the current value of $y < \delta$ and $y < \Delta$, we obtain:

$$\left.\begin{aligned} &\frac{\mathrm{d}\theta_y}{\mathrm{d}x} + \left(\frac{2 + H_y}{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x} + \frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x} + \frac{1}{\rho_1}\frac{\mathrm{d}\rho_1}{\mathrm{d}x}\right)\theta_y - \frac{u_1 - u}{r\rho_1 u_1^2}\int_0^y \frac{\partial(r\rho u)}{\partial x}\,\mathrm{d}y \\ &\qquad = \frac{\tau_w}{\rho_1 u_1^2} - \left(1 - \frac{y\cos\beta}{r}\right)\frac{\tau_y}{\rho_1 u_1^2} \end{aligned}\right\} \quad (13)$$

$$\left.\begin{aligned} &\frac{\mathrm{d}\varphi_y}{\mathrm{d}x} + \left[\frac{1}{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x} + \frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x} + \frac{1}{\rho_1}\frac{\mathrm{d}\rho_1}{\mathrm{d}x} + \frac{1}{T_{01} - T_w}\frac{\mathrm{d}(T_{01} - T_w)}{\mathrm{d}x}\right]\varphi_y \\ &- \frac{T_{00} - T_0}{\rho_1 u_1 (T_{01} - T_w)}\int_0^y \frac{\partial}{\partial x}(r\rho u)\,\mathrm{d}y \\ &= \frac{1}{c_p \rho_1 u_1 (T_{01} - T_w)}\left(q_w - \frac{r_y}{r}q_y\right) \end{aligned}\right\} \quad (14)$$

where

$$\theta_y = \int_0^y \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right)\left(1 - \frac{y\cos\beta}{r}\right)\mathrm{d}y$$

is the thickness of impulse loss for a fixed value of y;

$$\delta_y^* = \int_0^y \left(1 - \frac{\rho u}{\rho_1 u_1}\right)\left(1 - \frac{y\cos\beta}{r}\right)\mathrm{d}y$$

is the thickness of displacement for a fixed value of y;

$$\varphi_y = \int_0^y \frac{\rho u}{\rho_1 u_1}\left(\frac{T_{00} - T_0}{T_{01} - T_w}\right)\left(1 - \frac{y\cos\beta}{r}\right)\mathrm{d}y$$

is the thickness of energy loss for a fixed value of y.

Using experimental values, entering equations (13) and (14), varying y from zero to δ, and from zero to Δ, the coefficients of resistance and the specific heat flows were calculated. The results are given as diagrams $c_{fw} - c_{fy} = f(y)$ and $[(q_y - q_w)/c_p] = f(y)$ for different sections of experimental portions for different flow rates of heated air.

The diagrams of the distribution of tangential stresses and specific heat flows in a boundary layer were used for determining the turbulent transfer coefficients of impulses and heat A_τ and A_q using equations:

$$A_\tau = \frac{\tau}{\mathrm{d}u/\mathrm{d}y}; \quad (15)$$

$$A_q = \frac{q}{c_p(\mathrm{d}\tau/\mathrm{d}y)}. \quad (16)$$

The distribution of heat transfer coefficients in a boundary layer is presented by the equation $A_q/A_\tau = f(y)$ for all parts of experimental portions at characteristic flow rates of cooled air.

4. RESULTS OF EXPERIMENTAL INVESTIGATIONS

The distribution of velocities and of temperatures at the section of a turbulent boundary layer for different values of the form parameter of the pressure gradient Γ is given in Fig. 3.

On the diagram the curve corresponds to a $\frac{1}{7}$ power law for velocity distributions.

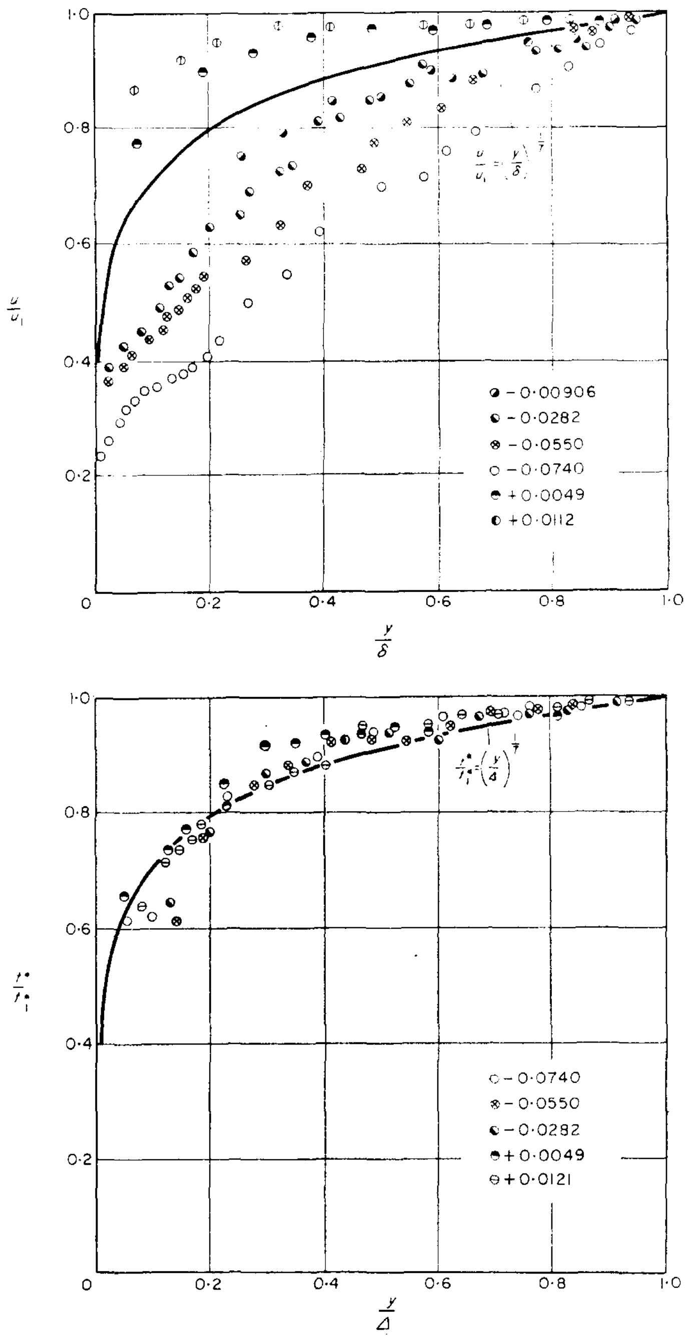

FIG. 3. The influence of pressure gradient on the distribution of velocities and temperatures in a boundary layer.

From diagrams of the velocity, and temperature distribution it can be seen that with increasing pressure gradient the velocity profiles in a diffuser region become less filled. In concentrator channels the velocities profiles are fuller the greater the pressure gradient. Our experiments showed that the same trend (but to a lesser extent) is also observed with respect to the temperature distribution.

Comparison of the diagrams of velocity and temperature distributions in a boundary layer shows that for gas flows with positive pressure gradients similarity of velocity and temperature fields do not obtain. In particular, considerable disturbances in the similarity of these velocity and temperature fields is observed in a pre-separation region.

In order to determine whether the parameter Γ is a suitable form parameter for studying the behaviour of a turbulent boundary layer for gradient gas flow with heat transfer, a diagram was plotted giving the dependence of the value $H = \delta^*/\theta$ (at the calculation of δ^* in Dorodnitsyn's variables) characterizing the form of the velocity profile from Γ (Fig. 4). This diagram shows that the ratio δ^*/θ simply depends upon Γ, the function of $H = \delta^*/\theta = f(\Gamma)$ being independent of the Reynolds number. Therefore, the parameter Γ is really a suitable form parameter.

The relation between values H and Γ is expressed by the equation:

$$H = 1{\cdot}47\,(1 - 4{\cdot}55\Gamma). \qquad (17)$$

Considering the diagram (Fig. 4), we notice that the separation of the boundary layer is not observed for values of the form parameter Γ exceeding twice the separation values of $\Gamma = 0{\cdot}06$ by Buri [1]. Consequently, the values of H obtained in experiments, were of the order 2·5 which approaches the separation values of H recommended by Doenhoff and Tetervin [4]. The essential delay of the separation point in comparison with the motion of an incompressible liquid can be explained by the stabilizing influence of cooling diffuser walls as well as by the axisymmetrical nature of gas motion under these conditions. The possible influence of the roughness of a channel surface was practically eliminated, since the diffuser surface was thoroughly finished. From the physical viewpoint the wall cooling causes, as is known, an increase in velocity gradients near the walls, making the velocity profile fuller, and all this involves the delay of the separation point of the boundary layer.

The noticeable scatter of experimental values on the diagram $H = f(\Gamma)$ can be explained by the influence of different cooling conditions. Evidently the determination of H by the thickness of displacement in Dorodnitsyn's variables does not make it completely possible to eliminate the influence of heat transfer.

Equation (17) can be used for practical calculations of a dynamic turbulent boundary layer for a range of the temperature factor $\bar{T}/\bar{T}_w$ from 0·5 to 1.

In order to obtain a closed system of equations which determine relations between pfoperties of the dynamic boundary layer, it is necessary to substitute an additional form parameter for a

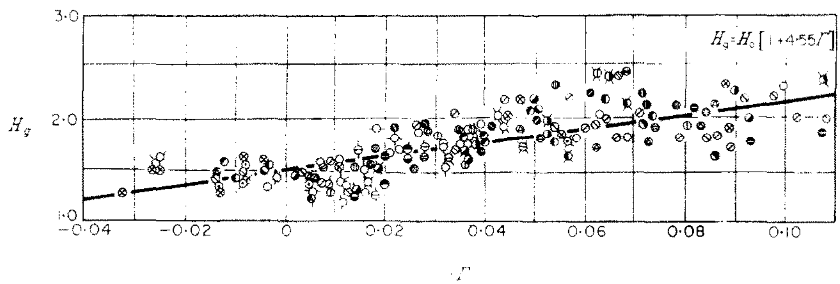

FIG. 4. The universal relation between form parameters of a dynamic boundary layer.

$$H_g = \frac{\delta_g^*}{\theta} \text{ and } \Gamma = \left(-\frac{\theta}{\rho u_1^2}\frac{dp}{dx}\right) Re_\theta^{1/4}.$$

turbulent boundary layer with a pressure gradient characterizing the friction law. For a turbulent gas flow without pressure gradient the friction law in its simplest case corresponding to the $\frac{1}{7}$ power law for the velocity distribution is expressed as follows:

$$\zeta = \frac{\tau_w}{\rho_1 u_1^2} Re_\theta^{1/4}. \qquad (18)$$

For a flat plate, as is known, ζ is a constant and equals 0·0128.

compressible liquid with longitudinal pressure gradient.

Having obtained experimental data on the resistance for heated air flow in concentrators and diffusers with cooled walls, we tried to find out whether it is possible to preserve the form parameter ζ in such a form as did Buri.

As it turned out, the considerable scatter of experimental values on the plot $\zeta = f(\Gamma)$ is explained by the influence of heat transfer. This influence is determined by the substitution of

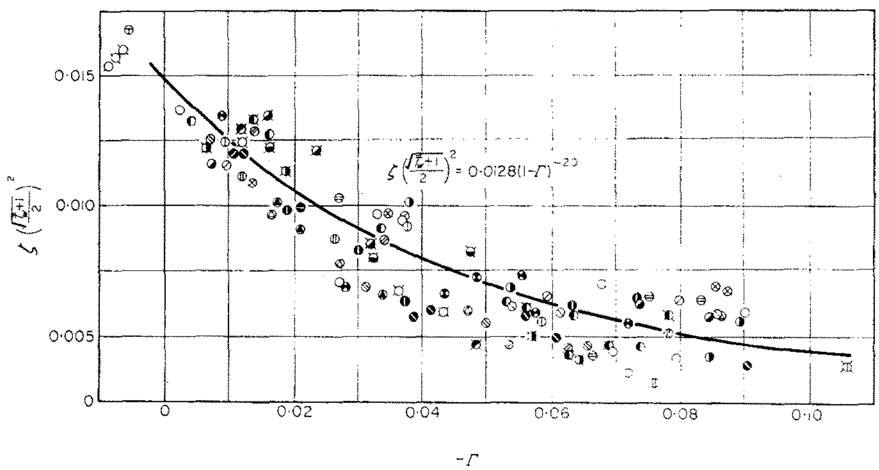

FIG. 5. The universal relation between the friction coefficient $\zeta = (c_f/2)Re_\theta^{1/4}$ and the form parameter Γ under non-isothermal conditions.

Since the pressure gradient greatly influences the tangential friction stress on a wall, it is obvious that in this case the value of ζ cannot be a constant, but a function of the pressure gradient, and that it equals zero at the separation point of the boundary layer. On the basis of experimental data, made by Nikuradse [12], Buri showed that for a gradient flow of an incompressible liquid without heat transfer the value of ζ, determined by equation (18), is a single-valued function of the pressure gradient form parameter Γ, this function as well as that of $H(\Gamma)$ being independent of the Reynolds number. Therefore the parameter ζ, expressed by equation (18), appeared to be a really suitable form parameter for calculating the turbulent boundary layer for isothermal flow of an incompressible liquid with longitudinal pressure gradient.

the temperature factor $\bar{T}_w$ into the functional dependence $\zeta = f(\Gamma)$.

For a first approximation the influence of heat transfer on ζ is taken into account by using a limit formula obtained elsewhere [13]. The dependence of $\zeta\{[\sqrt{(\bar{T}_w)} + 1]/2\}^2$ upon Γ while determining ζ and Γ by equations (18) and (9) is shown in Fig. 5.

On applying a correction for heat transfer by using the limit formula, the scatter of the experimental points was decreased, but still remained considerable, and this is explained by the fact that the formula was obtained for a gas flow of uniform stream velocity.

In Fig. 5 the experimental points lie along a satisfactory curve, the equation of which is as follows:

$$\zeta\left(\frac{\sqrt{(\bar{T}_w)}+1}{2}\right)^2 = 0{\cdot}0128\,(1-\Gamma)^{-20} \qquad (19)$$

and hence

$$\zeta = 0{\cdot}0128\left(\frac{2}{\sqrt{(\bar{T}_w)}+1}\right)^2\frac{1}{(1-\Gamma)^{20}}. \qquad (20)$$

Taking into account equation (18), we get the law of resistance

$$\frac{c_f}{2} = \frac{0{\cdot}0128}{Re_\theta^{1/4}}\left(\frac{2}{\sqrt{(\bar{T}_w)}+1}\right)^2\frac{1}{(1-\Gamma)^{20}}. \qquad (21)$$

Experimental data, including the limit of the temperature factor change from 0·5 to 1·0 and values of the form parameter Γ from zero to 0·12, are used for plotting the diagram in Fig. 5.

The dependence of the form parameter H on that η substituted by Gruschwitz [2], is given in Fig. 6.

$$\eta = 1 - \frac{u^2(\theta)}{u_1^2}. \qquad (22)$$

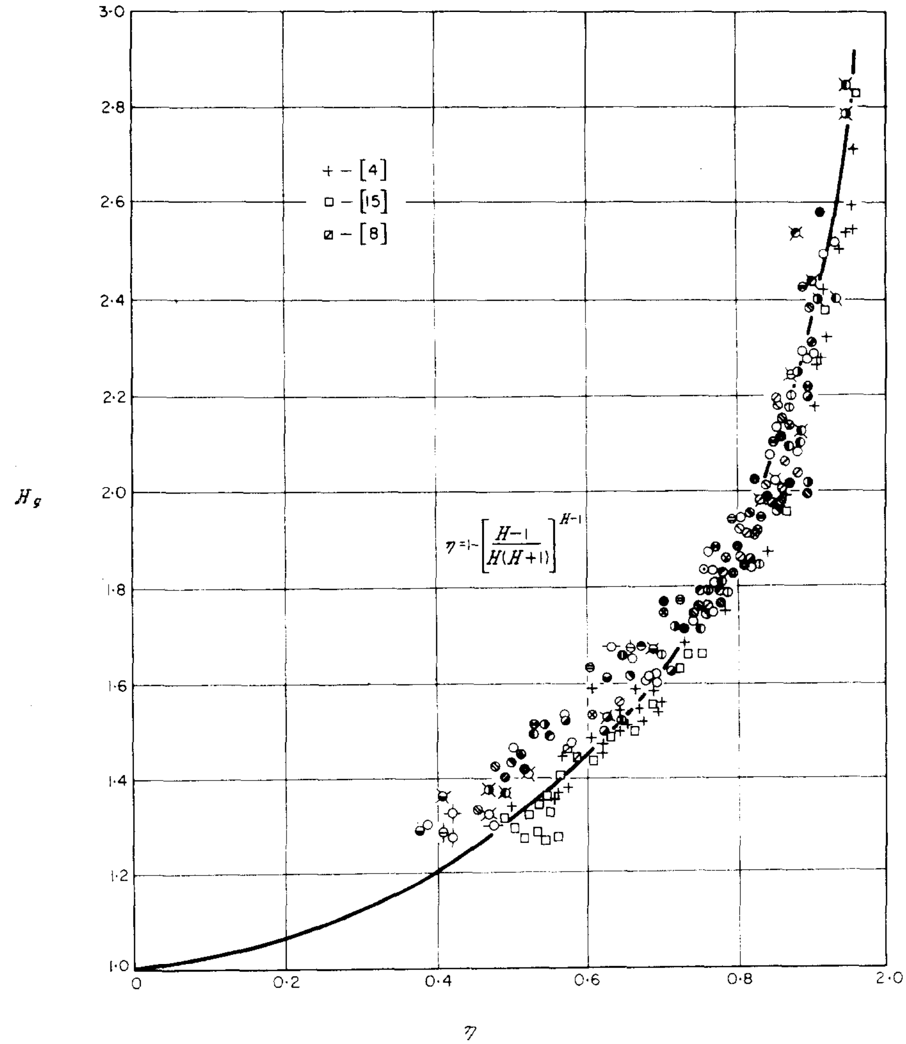

FIG. 6. The universal relation between form parameters

$$H_g = \frac{\delta_g^*}{\theta} \quad\text{and}\quad \eta = 1 - \frac{u^2(\theta)}{u_1^2}.$$

The form parameter is connected with that of $H = \delta^*/\theta$ by the equation

$$\eta = 1 - \left(\frac{H-1}{H(H+1)}\right)^{H-1} = 1 - \left(\frac{\theta}{\delta^*}\right)^{2/n} \quad (23)$$

obtained with the power law of the velocity distribution:

$$\frac{u}{u_1} = \left(\frac{y}{\delta}\right)^{1/n}$$

where $u(\theta)$ is the velocity in the boundary layer at a distance from a wall $y = \theta$;

$$n = \frac{2}{H+1}.$$

Besides our experimental values, the experimental values made by Gruschwitz [2], Nikuradse [12], Kehl [14], Doenhoff and Tetervin [4], Brebner and Bagley [9], Schubauer and Klebanoff [11] were plotted in Fig. 6. From the plot it can be seen that the experimental values of Gruschwitz, Nikuradse and Kehl lie well on the Gruschwitz curve (equation 23) for a range of small values of the form parameter H. Our data as well as the experimental values obtained by the other investigators mentioned include a region of great values of form parameters H and η in which a noticeable deviation from the dependence recommended by Gruschwitz is observed.

In order to make more precise calculations it is suggested that a smoothed curve through our experimental points be used. The experimental values, grouped near one curve with relatively small scatter, confirm the choice of η as a form parameter for determining the separation point of a boundary layer.

5. THE CALCULATION OF A DYNAMIC BOUNDARY LAYER

Using equations (17) and (20), it is possible to obtain the method of calculating an axisymmetrical turbulent boundary layer for gas flow.

Take equation (1) as follows:

$$\frac{dRe_\theta}{d\bar{x}} + (H+1)\frac{Re_\theta}{u_1}\frac{du_1}{d\bar{x}} + \frac{Re_\theta}{r_1}\frac{dr_1}{d\bar{x}} = \frac{\tau_w Re_L}{\rho_1 u_1^2} \quad (24)$$

where

$$\bar{x} = \frac{x}{L}; \quad Re_L = \frac{\rho_1 u_1 L}{\mu_{00}};$$

L is the characteristic linear dimension; μ_{00} is the dynamic viscosity coefficient of potential flow at the temperature of deceleration.

Applying equation (18) we obtain:

$$\frac{dRe_\theta}{d\bar{x}} + (H+1)\frac{Re_\theta}{u_1}\frac{du_1}{d\bar{x}} + \frac{Re_\theta}{r_1}\frac{dr_1}{d\bar{x}} = \zeta^{-1/4} Re_L \quad (25)$$

Multiplying this equation by $Re_\theta^{1/4}$ and taking into account equation (9), we get:

$$\frac{d(Re_\theta^{5/4})}{d\bar{x}} + \frac{5}{4}\frac{Re_\theta^{5/4}}{r_1}\frac{dr_1}{d\bar{x}} = (5/4)\, Re_L\, [\zeta - (H+1)\Gamma]. \quad (26)$$

Designate

$$F(\Gamma) = (5/4)\, [\zeta - (H+1)\Gamma]. \quad (27)$$

The dependence of the function $F(\Gamma)$ on the form parameter Γ is given in Fig. 7. The values of $F(\Gamma)$, calculated on the basis of the values of ζ, H and Γ obtained in our experiments, lie on a straight line the equation of which has the form

$$F(\Gamma) = a - b\Gamma \quad (28)$$

where a and b are constants; from Fig. 7 it follows that $a = 0{\cdot}016$, $b = 3{\cdot}55$.

Thus, although the values ζ and H essentially depend on heat transfer, the latter does not influence the function $F(\Gamma)$.

Taking into account equation (28) and substituting Γ for its expression (9), we write down equation (26) as follows:

$$\frac{d(Re_\theta^{5/4})}{d\bar{x}} + \frac{5}{4}\frac{Re_\theta^{5/4}}{r_1}\frac{dr_1}{d\bar{x}} + b\frac{Re_\theta^{5/4}}{u_1}\frac{du_1}{d\bar{x}} = aRe_L. \quad (29)$$

The integral of equation (29) is:

$$Re_\theta^{5/4} = \frac{a}{r^{1\cdot25}u_1^b}\left(\int_{\bar{x}_s}^{\bar{x}} Re_L r^{1\cdot25} u_1^b \, d\bar{x} + c\right)$$

or

$$Re_\theta = \left[\frac{0{\cdot}0160}{r^{1\cdot25}u_1^{3\cdot55}}\left(\int_{\bar{x}_s}^{\bar{x}} Re_L r^{1\cdot25} u_1^{3\cdot55}\, d\bar{x} + c\right)\right]^{0\cdot8} \quad (30)$$

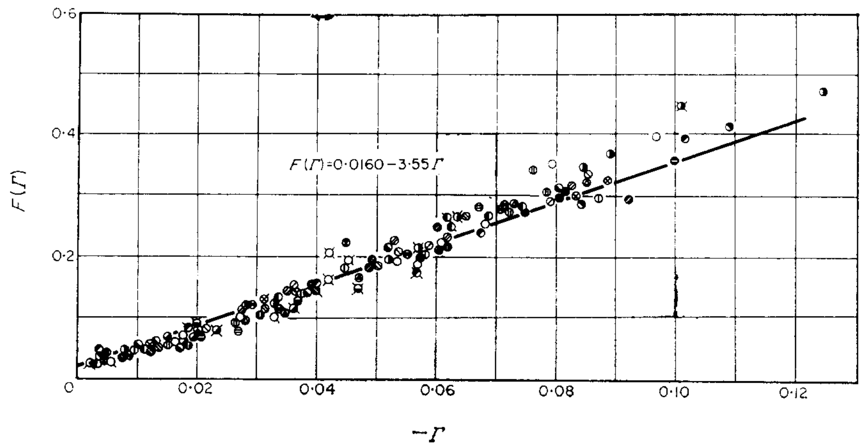

FIG. 7. The universal function $F(\Gamma) = 0{\cdot}0160–3{\cdot}55\Gamma$.

where c is the constant of integration

$$c = \left[\frac{Re_\theta^{1\cdot25} r^{1\cdot25} u_1^{3\cdot55}}{0{\cdot}0160}\right]_{\bar{x}=\bar{x}_s}$$

$\bar{x}_s$ is the value of the dimensionless co-ordinate x for an initial point of a calculated turbulent boundary layer.

On the basis of the experimental data obtained as well as on their treatment and generalization which permits one to integrate the momentum equation, we recommend the following succession for the calculation of the dynamic boundary layer for turbulent gas flow in axisymmetrical channels with longitudinal pressure gradient and heat transfer.

(1) For given laws of change of the velocity and temperature for a potential flow and for the channel radius along the longitudinal co-ordinate x the Reynolds number Re_θ, combined with the thickness of impulse loss, is determined depending on x by equation (30), and, consequently, the dependence of the thickness of impulse loss θ upon x is determined by equation (30) as well.

(2) The value of the form parameter Γ as a function of x is calculated by equation (9).

(3) For a given law of the change of wall temperature along the co-ordinate x and for a known temperature recovery coefficient of a potential flow the resistance coefficient on a wall depending on x is determined by equation (21).

(4) The value of the form parameter H as a function of x is determined by equation (17).

The method of calculating proposed should be applied for a range of relatively small values of the temperature factor ($\bar{T}_w = 0{\cdot}5–1$).

Our next work will be devoted to the problem of the influence of intensive cooling or heating of a gas upon turbulent boundary layer properties at the gradient gas flow.

6. HEAT BOUNDARY LAYER CALCULATION

The heat boundary layer calculation is carried out using the integral energy equation and the experimental data on the temperature distributions in a boundary layer and on the temperatures of the channel wall.

Let us write down equation (2) as follows:

$$\frac{\mathrm{d}Re_\varphi}{\mathrm{d}\bar{x}} + \frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\bar{x}} Re_\varphi + \frac{1}{T_{01} - T_w}\frac{\mathrm{d}(T_{01} - T_w)}{\mathrm{d}\bar{x}} Re_\varphi = St\, Re_L \quad (31)$$

where

$$Re_\varphi = \frac{u_1 \varphi}{\nu_1}; \quad St = \frac{q_w}{c_p \rho_1 u_1 (T_{01} - T_w)}.$$

Equation (31) can be easily converted to the form:

$$\left.\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}\bar{x}}(rRe_\varphi^{5/4}) + \frac{1}{4}\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\bar{x}}(rRe_\varphi^{5/4}) \\ &+ \frac{5}{4}\frac{1}{T_{01}-T_w}\frac{\mathrm{d}(T_{01}-T_w)}{\mathrm{d}\bar{x}}(rRe_\varphi^{5/4}) \\ &= \frac{5}{4}rRe_L\, St\, Re_\varphi^{1/4}.\end{aligned}\right\} \quad (32)$$

The treatment of our experimental data on the temperature distribution in a boundary layer showed that the values of the product $St \,.\, Re_\varphi^{1/4}$ plotted as ordinates for the values of the form parameter of a heat boundary layer with the pressure gradient Γ_h equal to:

$$\Gamma_h = \frac{\varphi}{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x}Re_\varphi^{1/4} = \left(-\frac{\varphi}{\rho_1 u_1^2}\frac{\mathrm{d}p}{\mathrm{d}x}\right)Re_\varphi^{1/4} \quad (33)$$

lie on a straight line, parallel to the abscissa (Fig. 8). Thus the product $St \,.\, Re_\varphi^{1/4}$ does not depend on the pressure gradient. It is possible to take the average value of $St \,.\, Re_\varphi^{1/4}$ equal to 0·011 from the plot (Fig. 8).

Substituting the value 0·0110 for $St \,.\, Re_\varphi^{1/4}$ into equation (32), we get:

$$\frac{\mathrm{d}}{\mathrm{d}\bar{x}}(rRe_\varphi^{5/4}) + \left[\frac{1}{4}\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\bar{x}} + \frac{5}{4}\frac{1}{T_{01}-T_w} \times \frac{\mathrm{d}(T_{01}-T_w)}{\mathrm{d}\bar{x}}\right](rRe_\varphi^{5/4}) = 0{\cdot}0137\, rRe_L. \quad (34)$$

The integral of equation (34) is:

$$rRe_\varphi^{5/4} = \frac{0{\cdot}0137}{r^{1/4}(T_{01}-T_w)^{5/4}} \left[\int_{\bar{x}_s}^{\bar{x}} r^{5/4}(T_{01}-T_w)^{5/4}Re_L\,\mathrm{d}\bar{x} + c\right].$$

Consequently,

$$Re_\varphi = \left\{\frac{0{\cdot}0137}{r^{5/4}(T_{01}-T_w)^{5/4}} \left[\int_{\bar{x}_s}^{\bar{x}} r^{5/4}(T_{01}-T_w)^{5/4}Re_L\,\mathrm{d}\bar{x} + c\right]\right\}^{0{\cdot}8} \quad (35)$$

where c is the integration constant

$$c = \left[\frac{Re_\varphi^{5/4}r^{5/4}(T_{01}-T_w)^{5/4}}{0{\cdot}0137}\right]_{\bar{x}=\bar{x}_s}$$

$\bar{x}_s$ is the value of the dimensionless co-ordinate x at the starting point of a calculated heat turbulent boundary layer.

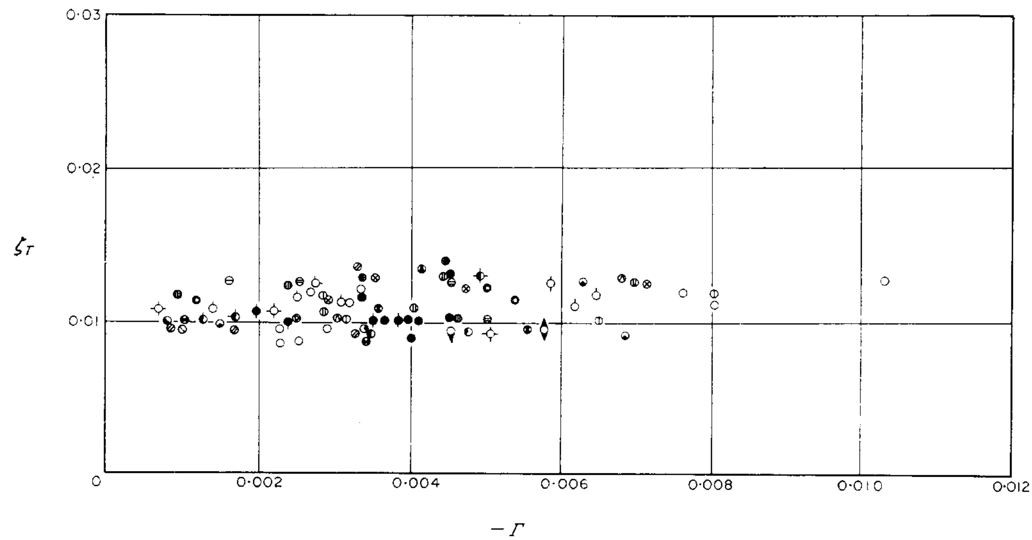

FIG. 8. The universal relation between form parameters of a heat boundary layer

$$\zeta_h = StRe_\varphi^{1/4} \quad \text{and} \quad \Gamma_h = \left(-\frac{\varphi}{\rho u_1^2}\frac{\mathrm{d}p}{\mathrm{d}x}\right)Re_\varphi^{1/4}.$$

For a constant wall temperature the equation

$$\frac{d}{d\bar{x}}(rRe_{\varphi}^{5/4}) + \frac{1}{4}\frac{1}{r}\frac{dr}{d\bar{x}}(rRe_{\varphi}^{5/4}) = 0{\cdot}0137\, rRe_L \quad (36)$$

should be integrated instead of equation (34).

Integrating this equation and solving it for Re_{φ}, we obtain

$$Re_{\varphi} = \left[\frac{0{\cdot}0137}{r^{5/4}}\left(\int_{\bar{x}_s}^{\bar{x}} r^{5/4} Re_L\, d\bar{x} + c\right)\right]^{0{\cdot}8}. \quad (37)$$

In order to determine the Stanton number depending on x, we use the equation

$$St = \frac{0{\cdot}0110}{Re_{\varphi}^{1/4}}. \quad (38)$$

The calculation of the heat boundary layer for turbulent gas flow in axisymmetrical channels with longitudinal pressure gradient is carried out in the following succession:

(1) For given laws of change of velocity and temperature in a potential flow and for given wall temperature and channel radius along the longitudinal x co-ordinate $Re_{\varphi}(x)$ and then the dependence of the thickness of energy loss $\varphi(x)$ are determined using equation (35) or (37). When the analytical law of velocity and temperature change for potential flow as well as that of wall temperature change is not predetermined, equations (35) and (37) may be numerically integrated.

(2) The values of the form parameter of the heat boundary layer Γ_h, depending on x, are determined by equation (33).

(3) The values of the Stanton number, depending on x, are calculated by equation (38).

(4) Heat flows on a channel wall are calculated using the equation:

$$q_w(x) = c_p \rho_1 u_1 (T_{01} - T_w)\, St. \quad (39)$$

(5) The relation between the form parameters of dynamic and heat boundary layers is determined from equations (9) and (33)

$$\frac{\Gamma_h}{\Gamma} = \left(\frac{\varphi}{\theta}\right)^{5/4}. \quad (40)$$

7. REYNOLDS ANALOGY OF HEAT AND IMPULSE TRANSFER

The experimental data obtained made it possible to analyse the problem of the validity of an analogy between processes of heat and momentum transfer for gradient gas flow. As is known, the equality of the coefficients of turbulent transfer of both impulses and heat as well as the identity of laws of changes of τ and q for a section of the boundary layer are the initial preconditions of the Reynolds analogy. The similarity of the velocity and temperature profiles for the boundary layer is a result of these preconditions.

The essential disturbance of the similarity of the velocity and temperature profiles in a boundary layer for gas flow with positive pressure gradient has been mentioned before. In order to find out reasons for the disturbance of the similarity of the velocity and temperature profiles, we checked the validity of the initial preconditions of the Reynolds analogy with gradient gas flow. The distribution of τ and q at a section of the boundary layer is given in Fig. 9 and was obtained with the help of the method of calculating mentioned in section 3. From the plot it can be seen that the laws of the change of τ and q for a section of the boundary layer essentially differ, this difference being the greater the closer the state of flow is to separation. While

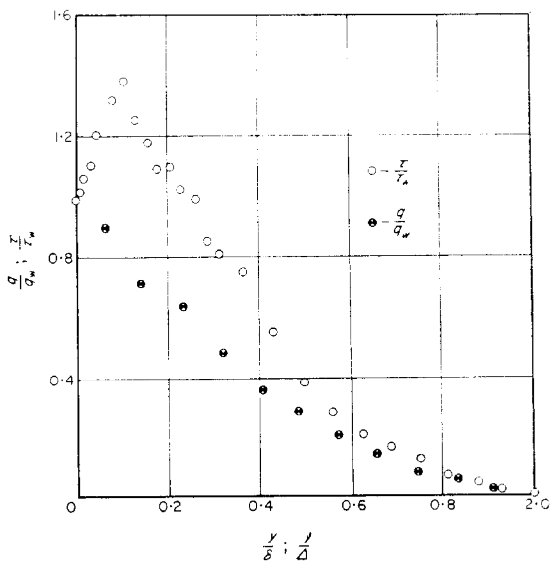

FIG. 9. The distribution of turbulent transfer coefficients of impulses and heat in the boundary layer.

removing from a plate the characteristic maximum with a further drop up to zero on the external boundary of the boundary layer is observed in the distribution of τ. The value of q continuously falls from the maximum value at the wall to zero at the outer boundary of the boundary layer. Thus, one of the preconditions of the Reynolds analogy is not confirmed by experiments with gradient gas flow.

In Fig. 10 the change of turbulent transfer coefficients both of impulses and heat A_τ and A_q for a section of the boundary is given, from which it is seen that the values of A_τ and A_q are similar near a wall. On moving away from the wall, the coefficient A_q exceeds A_τ, and at some distance from it A_q is approximately twice A_τ. Subsequently the ratio A_q/A_τ falls with increasing distance from the wall. Consequently, the second precondition of the Reynolds analogy (the equality between A_q and A_τ) is also not confirmed by experiments.

We obtained the regularity of change of A_q/A_τ for a section of the boundary layer which qualitatively agreed with experimental data obtained by Ludwieg [16], Elias [17], Fage and Fokner [18]. In particular, the latter obtained $A_q/A_\tau \approx 2$ on the basis of measurements of the temperature and velocity distribution in a free stream. One may assume that at some distance from the wall a mechanism of turbulent transfer of heat and impulses submits to regularities of free turbulence. From the physical viewpoint the influence of the free turbulence must prevail on approaching the separation point, since in this case the influence of the wall upon the properties of the turbulent boundary layer decreases.

The analysis carried out shows that the Reynolds analogy is not confirmed for gas flow with longitudinal pressure gradient. Consequently, all the semi-empirical methods of calculating a turbulent boundary layer, based on the application of the Reynolds analogy and, in particular, the Kalikhman method [5], cannot be applied to the gas flow with essentially positive pressure gradients. The inevitable result of these calculation methods is that $\tau_w = q_w = 0$ contradicts the physical essence of the process at the separation point of the boundary layer, and it is not confirmed experimentally.

CONCLUSIONS

New experimental data on the properties of dynamic and heat boundary layers for gas flow in axisymmetrical diffusers and in a concentrator with cooled walls were obtained.

The analysis of the experimental material obtained showed that existing calculation

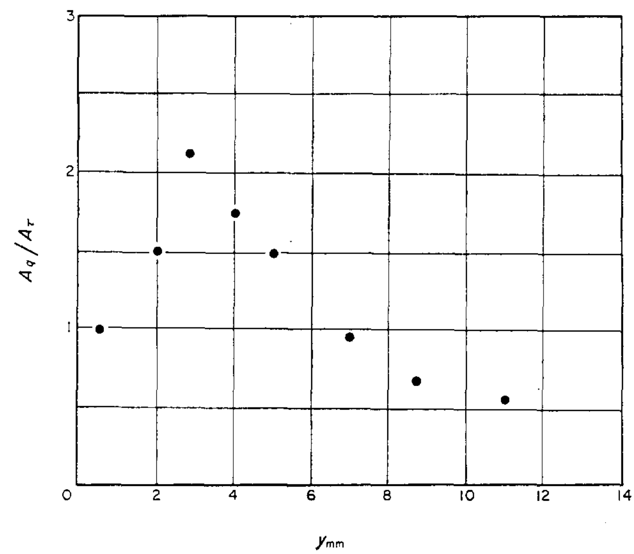

FIG. 10. The distribution of heat flow q and tangential stress τ in a boundary layer.

methods for turbulent boundary layers (Buri, Gruschwitz, Kalikhman) are not confirmed by experimental data under the conditions considered.

Experimental values of the form parameter Γ, corresponding to the separation point of a turbulent boundary layer, exceed the corresponding values of data by Nikuradse.

Experimental values of the form parameter ζ lie between those values obtained by the methods of Buri and Gruschwitz. The form parameter ζ, introduced by Buri for determining the law of friction for the gradient flow of an incompressible liquid without heat transfer, can be used to determine the law of friction at the gas flow with a pressure gradient and heat transfer when introducing a correction for heat transfer as well.

Experimental material on the dependence of the form parameter H upon η has been obtained by Gruschwitz for a region not previously considered.

A method of calculation of turbulent dynamic and heat boundary layers for gas flow in axisymmetrical channels with pressure gradient has been proposed.

From the existing methods of calculating turbulent boundary layers with a pressure gradient the methods based on the Reynolds analogy involve the most divergence from experiment results. Experiments showed that the essential disturbance of the similarity of velocity and temperature fields is observed for gradient gas flow. The ratio τ/q cannot be taken as constant over a section of a boundary layer. The ratio of turbulent transfer coefficients both of heat and impulses A_q/A_τ considerably change over a section of a boundary layer from ≈ 1 at the wall to ≈ 2 at the boundary of a boundary layer.

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Abstract—This paper presents the results of experimental investigation on the properties of a turbulent boundary layer for a flow of heated air in axisymmetrical diffuser and confuser channels having cooled walls. The experimental set-up made it possible to determine the influence of longitudinal pressure gradients and cooling intensity upon the properties of dynamic and heat boundary layers. The treatment and generalization of the experimental data were made in a parametric form. Dependences of form parameters, characterizing the form of the velocity and temperature profile, but also determining the law of friction and heat transfer from the longitudinal pressure gradient were found. These dependences from the longitudinal pressure gradient were found. These dependences are given in a form of design equations.

The extensive experimental material obtained made it possible to develop methods of calculation of a turbulent layer for gas flow with both longitudinal pressure gradient and heat transfer, which makes it possible to determine thicknesses of impulse and energy loss; thicknesses of dynamic and heat displacement; and the tangential stress and specific heat flow on a wall.

Moreover, the paper presents the results of an experimental investigation on the distribution of the tangential stress and specific heat flow as well as that of turbulent transfer coefficients of impulses and heat by a section of the turbulent boundary layer at the gradient flow.

Résumé—Cet article présente les résultats d'une recherche expérimentale sur les propriétés d'une couche limite turbulente dans le cas d'un écoulement d'air chaud dans des conduits de révolution, convergents ou divergents, dont les parois sont refroidies. Le dispositif expérimental utilisé a permis de déterminer l'influence des gradients de pression longitudinaux et de l'intensité du refroidissement, sur les propriétés des couches limites dynamique et thermique. On a exploité et généralisé les données expérimentales sous forme paramétrique. On a déterminé l'influence de paramètres de forme qui caractérisent l'allure des profils de vitesse et de température mais déterminent la loi de frottement et de convection à partir des gradients de pression longitudinale. L'influence de ces paramètres de forme a été évaluée à partir du gradient de pression longitudinale. L'ensemble des résultats obtenus permet de développer des méthodes de calcul d'une couche limite turbulente pour des écoulements de gaz avec des gradients de pression et de transmission de chaleur, en sorte qu'il est possible de calculer les épaisseurs de quantité de mouvement et d'énergie, les épaisseurs de déplacement dynamique et thermique, la contrainte tangentielle et le coefficient de convection à la paroi.

Cet article présente en outre les résultats d'une recherche expérimentale sur la distribution de la contrainte tangentielle, du coefficient de convection ainsi que le distribution du coefficient de transmission turbulente de la quantité de mouvement et de la chaleur dans une section de la couche limite turbulente.

Zusammenfassung—Die Ergebnisse experimenteller Untersuchungen von turbulenten Grenzschichten werden hier angegeben. Diese Untersuchungen erstreckten sich auf Luft, die in achsensymmetrisch sich erweiternden bzw. sich verengenden Kanälen mit kalten Wänden strömte. Die experimentelle Anordnung liess den Einfluss des Längsdruckgradienten und der Kühlintensität auf die Eigenschaften der hydrodynamischen und thermischen Grenzschicht erkennen. Die ermittelten Daten wurden in Parameterform verallgemeinert. Ein Formparameter, der sowohl die Form des Geschwindigkeits- und Temperaturprofils charakterisiert, als auch bestimmend für das Reibungsgesetz und den Wärmeübergang ist, zeigte sich vom Druckgradienten in Längsrichtung abhängig. Diese Abhängigkeit konnte gefunden und in Gleichungen angegeben werden.

Die umfangreichen Ergebnisse erlaubten die Entwicklung von Rechenmethoden für turbulente Grenzschichten in Gasströmungen mit Längsdruckabfall und Wärmeübergang. Danach können bestimmt werden: die Dicken für die Impuls- und Energieverluste, die hydrodynamischen und thermischen Verdrängungsdicken, die Schubspannungen und der spezifische Wärmefluss an der Wand.

Weiter sind Versuchsergebnisse angegeben für die Verteilung der Schubspannung und des spezifischen Wärmeflusses, sowie turbulente Austauschgrössen für Impuls and Wärme über den Querschnitt der turbulenten Grenzschicht bei Längsdruckabfall.